Understanding Resolution

Diffraction and The Airy Disk,
Dawes Limit & Rayleigh Criterion

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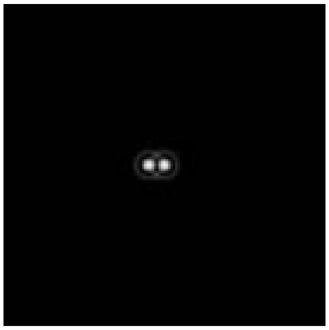


These explanations of terms are based on my understanding and application of published data and measurement criteria with specific notation from the credited sources. Noted paragraphs are not necessarily quoted but may be summarized directly from the source stated. All information if not noted from a specific source is mine. I have attempted to be as clear and accurate as possible in my presentation of all the data and applications put forth here. Although this article utilizes much information from the sources noted, it represents my opinion and my understanding of optical theory. Any errors are mine. Comments and discussion are welcome.

Clear Skies, and if not, Cloudy Nights.
EdZ
November, 2003.

Introduction: Diffraction and Resolution

All lenses or mirrors cause diffraction of light. Assuming a circular aperture, the image of a point source formed by the lens shows a small disk of light surrounded by a number of alternating dark and bright rings. This is known as the diffraction pattern or the *Airy pattern*. At the center of this pattern is the Airy disk. As the diameter of the aperture increases, the size of the Airy disk decreases. The aperture and the size of the Airy disk determine the limits of the scope to resolve two close point sources. Although there are many other factors involved, if no other aberrations limit the ability of the lens, then the lens is said to be diffraction limited, meaning limited in resolution only by the diffraction pattern it produces.

Resolution in optical instruments is dependant on the aperture of the lens or mirror and the wavelength of the light observed. Resolution is independent of focal length or magnification, however it is dependant on the magnitude and color of the stars observed. In telescopes we use the angular value for resolution, usually reported in arc seconds.

Suiter: “The Rayleigh resolution criterion is met when the separation of the two objects is precisely at the radius of the theoretical Airy disk. In other words, the second star is placed on the valley between the first star’s central disk and the first diffraction ring.”

Diffraction of light produces the *Airy pattern,* which appears as a central bright disk, the spurious disk, surrounded by concentric dark and bright rings. The radius to the point of minimum light within the first dark ring, the minima, is at the physical distance $r = 1.22 \lambda F$, or alternatively, the **angular distance $A = 1.22 \lambda / D$**. This gives the angular radius of the *Airy disk* in radians. Lambda (λ) is the wavelength of the light observed and D is the diameter of the scope in the same units. The formula is usually based on the wavelength of light to which we are most sensitive, that for yellow light, 5500 Angstroms or 550 nanometers. The Airy disk will be slightly larger for longer wavelengths, red light and slightly smaller for shorter wavelengths, blue light.

The Airy disk radius is measured from the midpoint of the central diffraction disk to the minimum of the first diffraction interspace. The central diffraction disk, sometimes confusingly referred to as the Airy disk, is somewhat smaller than the true Airy disk. As aperture increases, the Airy disk gets smaller and hence a larger lens has a greater resolving power. However the Airy disk is always the same size for a given aperture at a given wavelength of light. There is a broad array of published information that is either not completely defined or inaccurately defined and can lead to confusion or a misinterpretation concerning this topic.

There are numerous texts that claim the Airy disk is the central disk and that the formula results in the radius of the central disk. This is not correct. Keep in mind that the correct formula for the Airy disk gives the radius to the center of the first minima within the first dark interspace, so the angular dimension of the bright central disk itself is a bit smaller. Sidgwick gives a very good explanation of the Airy disk formula.

This following is very important to understand resolvability of various point sources of different brightness. The Airy disk can be seen as made up of two parts, the central disk or spurious disk and a portion of the first dark ring or the first diffraction interspace. However, ignoring for the moment the effects of atmosphere, the appearance of the Airy disk is affected by two things. First, the Airy disk angular size is determined by aperture and the wavelength of the light observed. As aperture increases, angular size of the Airy disk decreases. Also, the size of the Airy disk varies as wavelength varies. Second, the magnitude of the observed star has an affect on the appearance of the Airy disk. A very bright star puts so much light into the central disk that the disk itself may take up 85% of the Airy disk diameter. The central disk of a faint star may take up less than 50% of the Airy disk diameter, leaving more dark space in the first diffraction interspace. A reasonable assumption is for moderately bright stars near 6th magnitude 50% to 60% of the Airy disk diameter is occupied by the visible central disk. Magnitude has a significant affect on the resolvability of close point sources.

One might make the assumption then it would be much easier to resolve fainter stars as the smaller central disks could be closer making observation of a clean separation apparent before they would overlap. However, the inability of the eye to see fainter light at some point overwhelms the benefit of the smaller central disk. So fainter stars do have a smaller central disk diameter and may be easier to split but at some not clearly defined point are more difficult to see.

Regardless of the type of scope in use, the Airy disk is dependant on aperture. Given equal aperture, the type of scope in use will not change the size of the Airy disk but it will determine what percentage of the light is put into the central visible disk and what remainder of the light is put into the first diffraction ring, the ring just outside the first diffraction interspace. While a refractor will put more light into the disk and less light into the first diffraction ring, an obstructed scope will put a little less light into the disk and more light into this first ring. This will help us understand why it is sometimes difficult using a reflector to observe a double with a separation that places the secondary directly on the first diffraction ring. When the first ring is bright enough and the secondary component of the pair faint enough, the star may be completely lost in the glow of the ring.

Knowledge of point source diffraction is necessary to understand the application of resolution to extended objects. While there are some simple conditions where point source diffraction may seem to explain resolution in extended objects, many extended object conditions are much more complex and the criteria explaining point source resolution is altered in some way. Contrast becomes a very important attribute affecting resolution in extended objects. In most cases resolution in extended objects is dramatically affected by our abilities of visual perception.

Visual Acuity, the ability of the eye to see the resolved image, can vary from one individual to another. Acuity can be tested on objects with all dimensions of separation, even wide objects that require only low magnification. Minimum acuity is the apparent size of the image magnified to a size necessary for the eye to just be able to see the resolved image in the focal plane. Many confuse or blend a discussion of acuity with resolution. It would be very difficult to discuss resolution without discussing acuity or the ability of the eye to perceive the resolved image. Many discuss limits of the eye’s acuity without a clear explanation. There is a much different average limit for bright daylight (~1 arcmin), night viewing (~3 arcmin), resolution threshold viewing (~4 arcmin) and faint star mag. 9+/- viewing (~5 arcmin) due to inefficiencies in the eye. Knowing your own visual acuity will help you determine the magnification needed to see resolved images.

It is a complete knowledge of all aspects mentioned that allow the observer to fully understand resolution and the limits it imposes on our equipment and our ability to see specific objects.

Common Diffraction Limit Criteria

Porcellino: “No amount of magnification will allow you to resolve a star that is beyond your telescope’s limit.”

Resolution as used in astronomy is commonly measured by angular measure in units of arc seconds. The Airy disk has the same linear size (more or less) for all scopes of a given focal ratio. The Airy disk has the same angular size for all scopes of a given aperture.

In telescopes, the most important component is aperture. Aperture controls maximum resolution. Maximum potential resolution determines the ability to resolve both point sources and fine detail in extended objects.

The two most commonly used criteria for measuring limits of resolution are Rayleigh limit and Dawes limit.

Rayleigh Limit = 5.45/D inches = 138/Dmm. Rayleigh Limit is a measure defining the limit at which two components can be clearly identified as separate components. It defines the distance between the centers of two Airy disks where the maximum of one is placed over the minimum of the other.

The Rayleigh limit of a 6”/150mm telescope is 5.45/6” or 138/150 = 0.91 arcseconds.

Rayleigh Limit is a measure that correlates to the wave nature of light. The correlation will be shown later.

Kitchin: The angular resolution is equal to the **Rayleigh** limit, where separation between two stars is considered as achieved when the stars are just touching.

When observing stars at the theoretical limit, for the condition “just touching” to be possible, the visible disk must be no more than 50% of the diameter of the Airy disk. This would be true only for moderately faint stars. In this case, the distance between the centers of the two Airy disks is equal to the diameter of each of the central bright disks and it has the radii of the two visible disks just touching. This is not to be misunderstood as a measure of a space between the bright edges of the two Airy disks.

Based on a strict interpretation of the Rayleigh limit, two point sources with a separation of less than 0.91 arcsec could not be separated with a 6” scope. This is not always true in the strictest sense, as some equipment may be capable of exceeding standard limitation criteria. In addition, there are many other factors to consider, not the least of which is atmospheric turbulence.

Dawes Limit = 4.56/D inches = 116/Dmm. Dawes Limit is the first point at which a double star is elongated enough to suspect the presence of two stars. Like Rayleigh, it is not a measure of separation required to see a black space between components.

Dawes limit for a 6” / 150mm telescope is 4.56/6 or 116/150 = 0.77 arcseconds.

Dawes is not a measure dependant on the wave nature of light. It was empirically determined to represent a point of minimum separation where a double can be noticed as two components. It is the first point at which a noticeable notch allows a determination that two components exist.

Dawes developed a constant empirically by comparisons of the performance of various telescopes of different apertures, having examined a vast number of double stars. The constant is expressed in inches of aperture and arcseconds of separation. Please refer to Jeff Medkeff's link for a quote from Dawes in the *Memoirs* of the Royal Astronomical Society.

Dawes limit is not used to indicate an achievable black space between two point sources. It is however a reliable determination of a limit at which two moderately bright close stars can be noticed as being two separate components.

Sparrow Limit: A web article published by Tom Licha discusses the Sparrow Limit, a limit even closer than Dawes. The Sparrow Limit, in all cases is very near one half the distance calculated by the Rayleigh Limit.

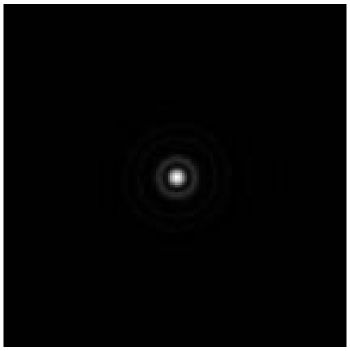
I have not discussed the Sparrow limit here in this article. For a good explanation of the Sparrow Limit and how it relates to resolution, please refer to Tom Licha's article. A web link is provided in the credits. Suiter, in his book "Star Testing Astronomical Telescopes", discusses and compares these three limits in his Appendix A.

The Airy Disk

In my research of the Airy disk formula, I noted numerous well-regarded astronomy references and several websites with published astronomy related formulae that refer to a shortened form of the formula for the Airy disk size. That shortened form in all cases was given as $A = 1.22/D$ where it is stated that A = arcsec and D = scope dia. in meters. This formula is incorrect. Further, some, not all, of the same books and websites imply that the Airy disk and Dawes Limit are even equivalent. There are in print a number of incorrect associations, wrong units used in formulae or a general lack of explanation to provide a full understanding for the presentation of the Airy disk formula.

Based on the definitions of Dawes Limit and Rayleigh Limit, it is clear that Dawes Limit and Airy disk size are not equivalent, but that Rayleigh Limit and Airy disk size are related and equivalent.

Do not confuse the definition of the Airy disk as the bright central dot in the diffraction pattern. This is really not correct and this term is very often confused in much of the literature in print. Diffraction produces a pattern called the Airy pattern, dominated by the Airy disk and then surrounded by a number of less bright diffraction rings. The Airy disk is measured out to the minimum of the first diffraction interspace. The central bright disk is correctly referred to as the spurious disk. There is no true measurement for the spurious disk itself.



I will refer to the spurious disk or central disk as the visible disk.

The measurement 5.45/D (see derivation below) based on the wavelength of light (and specific only to yellow light at 550 nanometers, the light to which we are most sensitive) is measured out to the first minima. As we move from the center of the visible disk out into the first dark interspace the minima occurs. The edges of the visible disk usually cannot be seen as the light falls off to zero towards the first minima. The dimension of the Airy disk varies with the wavelength of light, being larger for red light and smaller for blue light. Therefore, given equal magnitudes, it will be slightly easier to split two blue stars than two yellow stars and both are easier than two red stars.

Sidgwick: Resolving power is dependant on wavelength of the light observed and the diameter of the objective. The radius of the Airy disk is also referred to as the resolving power of the telescope.

Resolving power is not completely independent of the magnitude of the light observed and this will be explained later.

Beiser: The angular radius of the Airy disk out to the first minima is represented as:
$A = 1.22\ \lambda / D$, where
A in radians = 1.22 λ (Lambda) / D (Aperture)
A is the angular radius of the Airy disk measured in radians.
Lambda is the wavelength of light = 550 nm or 550 nanometers = 550 x 10^-9 meters.
Visible light is between 420 nm and 650 nm. We use 550 nm, the wavelength of yellow light.
D is the diameter of the aperture in meters. For a 150mm scope D = 0.15meters.

Then A = 1.22 x 550 x 10^-9 meters / 0.15meters = 0.0000045 radians
Converting radians to arcseconds,
then 0.0000045 radians x 360/2pi x 60 x 60 = 0.92 arcseconds.

The angular radius of the Airy disk for a point source resolved with a 150mm telescope observing yellow light at a wavelength of 550nm is 0.92 arc seconds.

Alternatively, Setting D equal to 1 inch then
A = 1.22 x 550 x 10^-9 meters / .0254meters = 0.0000264 radians
Converting radians to arcseconds,
then 0.0000264 radians x 360/2pi x 60 x 60 = 5.45 arcseconds.

The angular radius of the Airy disk for a point source resolved with a 1 inch telescope observing yellow light at a wavelength of 550nm is 5.45 arc seconds.
When D = 1 inch, A arcsec = 5.45/D. Therefore for any D, A arcsec = 5.45/Dinches.
This is the derivation of the constant for the Rayleigh Limit.

Kitchin: the angular resolution is equal to the Rayleigh limit, where separation between two stars is considered as achieved when the stars are just touching.

As mentioned before, for the condition “just touching” to be possible, the visible disk must be no more than

50% of the diameter of the Airy disk. This would be true only for moderately faint stars. Bright stars put greater light into the visible disk and very faint stars obviously put less light into the visible disk. However the size of the Airy disk remains constant for a given scope.

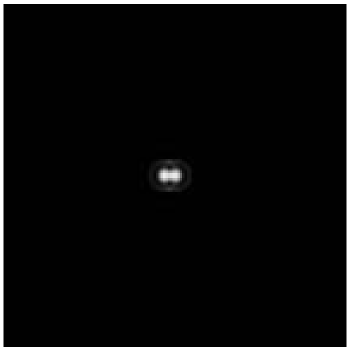
Repeated here, Rayleigh Limit resolution = 5.45/D inches = 138/Dmm. For a 6" scope, the Rayleigh limit is 5.45/6" = 0.91 arcseconds. This compares closely to the resolution calculated above using the formula for the wave nature of light. It varies only because the value 6" is nominal and does not correlate exactly to 150mm. 150mm/25.4 = 5.9 inches. 5.45/5.9 = 0.92 arcseconds.

Understanding Rayleigh and Dawes Limits

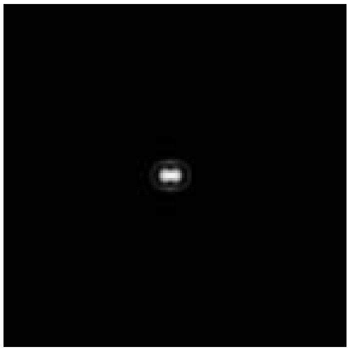
Rayleigh Limit = 5.45 / D inches (or 138 / D mm) is a measure of the ability of the scope aperture to split a double star.

Likewise**, Dawes Limit** = 4.56 / D inches (or 116 / D mm) is another measure.

Rayleigh Limit states you should be able to tell that a double is two stars if the centers of the diffraction disks of the two stars (commonly referred to as the Airy Disks, but see below) are separated by a dimension equal to the radius of the first diffraction interspace. That's the distance from the center of the Airy disk to the minimum of the space between the disk and the first diffraction ring. (This is important. I will refer to this a little further down). This calculation is directly tied to optics theory and the ability of a lens to resolve detail based on the wave nature of light. The limit of a lens to resolve is determined by the diameter of the lens and the wavelength of light. Take note that this limit, which has the centers of two disks separated by the radius of a disk may not provide for any black space between the two components.



Dawes limit was determined by actual field-testing of many and varied double stars. It states a limit for seeing a double star as two components when the centers of the two components are separated by a dimension defined by 116/Dmm. Similar to Rayleigh, this limit allows you to see a notch, not a complete black space, between the two components. But Dawes limit states you can see closer doubles than Rayleigh limit would indicate. Close doubles observed at the Dawes limit might be recorded as elongated or notched, but usually not separated.



It is true that you can tell there are two components to a double before you have reached a point where they are completely split with a black space between them. When observing doubles I keep my notes for the various eyepieces, using terms something like elongated, elongated pointed, notched, barely touching, thin black line, clear black space.

Generally, it is held that Rayleigh and Dawes should only be applied to equal 6th magnitude doubles. It will be shown that resolving limits are not independent of magnitude. My understanding is that although Dawes performed his testing on many and varied doubles, the stated limit is simply an average of his various results. Although all else here is commonly accepted, this averaging explanation warrants further reading. Some very good telescopes are capable of exceeding both of these limits. Conversely, some lesser quality scopes will not be able to even reach these limits. But these are good indications of what a good telescope should be able to see.

A list of doubles for testing Rayleigh or Dawes limit in scopes from 50mm to 250mm+:
First listing of Primary - Secondary / separation is from Sky Catalogue 2000.0 Volume 2,
designated with o if orbital elements are published. If o, sep. shown <decreasing or >increasing based on orbital elements for Visual Binaries published in Sky Catalogue 2000.0 Volume 2.
Second listing is from the USNO Washington Double Star database accessed from the internet.
Third listing if entered is from USNO 6th Orbit Catalogue. Although only minor change from current will occur in most cases, I selected the elements for the year 2004.

		mag1-mag2 / rho theta						
Name	Con	Prim-Sec / sep	PA	Prim-Sec / sep	PA	Prim-Sec / sep / PA	RA +/- Dec	Note
		SkyCatalogue2000		WDS (most 2002)		6th Orb Cat for 2004		
11 BC	Mon	5.2-6.1 / 2.8”	106	5.0-5.3 / 2.9”	108		RA06h29 –07 00	Σ 919
4 e1	Lyr	5.0-6.1 / 2.6”<	357o	5.0-6.1 / 2.5”	352	4.67-5.81/2.55”</349	RA18h44 +39 40	north
5 e2	Lyr	5.2-5.5 / 2.3”>	094o	5.2-5.4 / 2.3”	082	4.6-4.8 / 2.35”>/081	RA18h44 +39 40	south
21Mu	Dra	5.7-5.7 / 1.9”>	002o	5.7-5.7 / 2.2”	016	4.9-5.0 / 2.26>/013	RA17h05 +54 28	Σ 2130
16 z	Aqr	4.3-4.5 / 2.2”	188	4.3-4.5 / 1.9”	183		RA22h29 +00 01	Σ 2909
33	Ori	5.8-7.1 / 1.8”	027	5.7-6.7 / 1.9”	028		RA05h31 +03 18	Σ 279
113 a	Psc	4.2-5.2 / 1.8”<	269o	4.1-5.2 / 1.8”	271	3.8-4.9 / 1.8</269	RA02h02 +02 46	Σ 202
53	Aqr	6.4-6.6 / 3.1”	334	6.3-6.4 / 1.6”	014		RA22h26 –16 45	S,h 345
10 h	Oph	4.2-5.2 / 1.5"	033o	4.2-5.2 / 1.6”	031	3.82-4.85/1.45”/032	RA16h31 +02 00	Σ 2055
69 t	Oph	5.2-5.9 / 1.6”<	285o	5.3-5.9 / 1.5”	282	4.77-5.44/1.70</283	RA18h03 –08 11	Σ 2262
57	Cnc	6.0-6.5 / 1.4”	316	6.1-6.4 / 1.5”	312		RA08h54 +30 35	Σ 1291
48 e	Ari	5.2-5.5 / 1.5”	208	5.2-5.6 / 1.4”	209		RA02h59 +21 20	Σ 333
Pi	Aql	6.1-6.9 / 1.4”	110	6.3-6.8 / 1.4”	110		RA19h49 +11 49	Σ 2583

Σ 749	Tau	6.4-6.5 / 1.0”	333	6.5-6.5 / 1.1”	323		RA05h37 +26 55 3° se B Tau
36	And	6.0-6.4 / 1.0”	352o	6.1-6.5 / 0.9”	313	5.46-5.87/0.97”/315	RA00h55 +23 38 Σ 73
16 z	Cnc	5.6-6.0 / 0.9”>	065o	5.3-6.3 / 0.9”	072	5.6-6.0 / 0.93>/061	RA08h12 +17 40 Σ 1196
16	Vul	5.8-6.2 / 0.8”	115	5.8-6.2 / 0.8”	123		RA20h02 +24 56 OΣ 395
30	Boo	4.5-4.6 / 0.7”<	298o	4.5-4.6 / 0.7”	296	4.46-4.55/0.72”</298	RA14h41 +13 44 Σ 1865
8 y	Sex	5.6-6.0 / 0.6”	053o	5.4-6.4 / 0.5	059	5.07-5.55/0.61”/056	RA09h52 –08 06 AC 5
14 h	Cas	5.5-5.8 / 0.6”	194o	5.3-5.6 / 0.4"	199	4.73-4.77/0.44”</198	RA00h32 +54 31 OΣ 12
51	Aqr	6.5-6.5 / 0.3”	155o	6.5-6.6 / 0.3”	056	5.79-5.96/0.36”>/048	RA22h24 +04 50 B 172

The TV85, Televue’s 85mm apochromatic refractor, calculates to Rayleigh and Dawes limits of, Ray. Lim. = 138/85 (5.45/3.35) =1.63arcsec and Daw. Lim. = 116/85 (4.56/3.35) =1.36arcsec. A good double to test the ability to achieve Dawes limit is one that is nearly equal in magnitude, and it is neither too bright nor too faint. Pi Aquilae has components of magnitude 6.3 and 6.8 at a separation of 1.4 arc seconds (SC2000.0 indicates 6.1 and 6.9 at 1.4 arcsec).

Pi Aql 6.1-6.9 / 1.4” 110 6.3-6.8 / 1.4” 110 RA19h49 +11 49 Σ 2583

TV85	6UOx2TV	200x	elongated east west
TV85	5UOx2TV	240x	dark view, suspected f* following

and on another night,

TV85	5UO	120x	tiny, but elongation definite
TV85	8TVx2TV	150x	elongated point f* following
TV85	5UOx2TV	240x	clearly see b*f* but they still touch

I was not able to completely split this double to a black space, but I was able to identify it as a double at several magnifications. So I did reach Rayleigh and Dawes limits. But is this scope capable of exceeding these limits? I’ll explain that in the next section.

Seeing a Black Space Between Components

Now back to the passage I referred to as important. Let me further explain. First I will repeat what I said earlier. Rayleigh Limit states you should be able to tell a double is two stars if the centers of the diffraction disks of the two stars are separated by a dimension equal to the radius of the first diffraction interspace. That’s the radius from the center of the Airy disk to the minimum light within the space between the disk and the first diffraction ring.

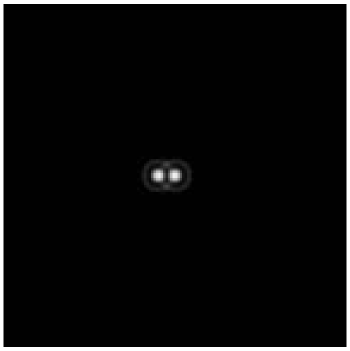
What if I want to see a double star with at least a thin black space between the components? What are my limits? What should I expect of my scope?

Based on the definition above, Rayleigh limit is a measure of a radius. It is the measure from the center of the bright central dot, or the visible disk, out to the minimum of the “black space” between the visible disk and the first bright diffraction ring that surrounds the Airy disk. If you want to see two stars as completely separated with a thin black space between them it is necessary for the centers of the Airy disks of the two components to be separated so this “black space” overlaps and becomes visible between them. That separation dimension for bright stars is approximately equal to two radii or the diameter of the Airy disk. Rayleigh Limit for my TV85mm scope is 138/85 or 1.6 arcseconds radius. Therefore the Airy disk diameter is 2 x 1.6” or 3.2 arcseconds.

The visible disk itself is slightly smaller than the Airy disk dimension since the Airy disk is measured out to the minima of the first dark space. The visible disk varies with magnitude, but it's about 85% of the Airy disk dimension for a bright star. For a faint star less than 50% of the diameter of the Airy disk is occupied by the central bright disk. We'll use 60% for the following example.

Suppose, using the TV85, I chose to observe two stars (assuming 60% visible disks) with a measured separation at the calculated Rayleigh Limit of 1.6". The centers of the Airy disks will be 1.6" apart. The Airy disks for each are 1.6" x 2 = 3.2". The visible disks assuming 60% each measure 1.9" in diameter and each have a radius of 0.95". The visible disks with radius of 0.95" need to squeeze into a space only 1.6" wide. They would overlap by (0.95+0.95) – 1.6 = 0.3 arcseconds, nearly 20% overlap.

A separation of something more than the Rayleigh Limit is needed to have a black space between two stars. If Rayleigh Limit (5.45/Dinches) for my 85mm(3.35inches) scope is 1.6 arcseconds radius, then in order to cleanly split doubles in my example, I may need 2 x 1.6" or 3.2" diameter x 60% visible disk or 1.9 arcseconds separation to at least see a thin black space between them.



If the stars with separation at the Rayleigh limit of 1.6" were faint and had a visible disk just a hair less than 50% the diameter of the Airy disk, you would be able to see a thin line of separation.

Using a reasonable 60% bright central disk assumption, it requires stars separated by Rayleigh Limit times 1.2 (60/50), in this case 1.6" x 1.2 = 1.9" before you can see a split. For very bright stars with a central disk 85% the diameter of the Airy disk, it would require stars at separation of Rayleigh Limit times 1.7 (85/50) or 1.6" x 1.7 = 2.7" before you could see a split. Very bright and very faint stars will begin to impose their own sets of limitations on resolvability due to magnitude.

I was not able to split 10 (Lambda) Ophiuchi with mags 3.8 and 4.85 at a separation of 1.45".

10 h	Oph	4.2-5.2 / 1.5"	033o	4.2-5.2 / 1.6"	031	3.82-4.85/1.45"/032	RA16h31 +02 00 Σ 2055

TV85	6UOx2TV	200x	elongated slightly, f* north following
TV85	5UOx2TV	240x	still only elongated, no split

Even at 300x, this tight double was not cleanly split with the 85mm scope. It took 344x in a 5"SCT to see the central disks clearly separated and still the first diffraction rings overlapped.

I was able to completely split 69 (Tau) Ophiuchi mags 4.8 and 5.4 at 1.7" to a thin black line.

69 t Oph 5.2-5.9 / 1.6”< 285o 5.3-5.9 / 1.5” 282 4.77-5.44/1.70</283 RA18h03 –08 11 Σ 2262

TV85	7.5 Takx2TV	160x	two orbs seen but not separated
TV85	6UOx2TV	200x	split looked like a thin line

The 5”SCT shows 69 Oph just barely seen as two components at 183X and very clearly split at 230x with color in the two components. This gives an indication either it is much easier to see doubles when the magnitudes are closer or 69 Oph is truly a little wider, or both.

Considering these to be stars of moderate magnitude that fall in the 60% bright central disk range, these stars seem to be just beyond or at the limits of this 85mm aperture. By all indications 69Oph clearly seems to show the TV85 exceeding the Rayleigh Limit based on the above calculations.

Limits I have reached with several of my scopes include:

CR150 ref	7.5Ult x2Ult	340x	30 Boo	4.5-4.5 / 0.7”	two distinct disks but touch
CR150 ref	5UOx2TV	480x	16 Vul	5.8–6.2 / 0.8”	moments of split
CR150 ref	4UO	300x	54 h Cyg	4.8-6.1 / 0.9”	confirmed steady split
G5/125 sct	4UO	340x	54 h Cyg	4.8-6.1 / 0.9”	two airy disks constant but no split
G5/125 sct	5UO	274x	Pi Aql	6.1-6.9 / 1.4”	moments separated black, faint
G5/125 sct	8TVplsl	172x	57 Cnc	6.1-6.4 / 1.5”	split very close and small
G5/125 sct	7.5Ultima	180x	y Virgo	3.5-3.5 / 1.5”	split at 180x May01
TV85 ref	5UOx2TV	240x	10 h Oph	3.8-4.8 / 1.45”	elongated, noticed correct PA
TV85 ref	5Uox2TV	240x	57 Cnc	6.1-6.4 / 1.5”	notch apparent, suspect black line
TV85 ref	6UOx2TV	200x	69 t Oph	4.8-5.4 / 1.7”	split to thin black line
AT1010/78ref	7UOx2.2Ult	150x	53 Aqr	6.3-6.4 / 1.6”	moments suspect elong Nov03
AT1010/78ref	7UOx2.2Ult	150x	zeta Aqr	4.3-4.5 / 1.9”	thin sliver of black Nov03
Swift76 sn	7.5Takx2Vix	160x	zeta Aqr	4.3-4.5 / 1.9”	very thin sliver of black Nov03
Swift76 sn	17Plx2Vix	71x	e2 Lyra	4.6-4.8 / 2.35”	clearly split south double
KD60 ref	6CelVixortho	69x	e2 Lyra	4.6-4.8 / 2.35”	perfect diff disks barely split

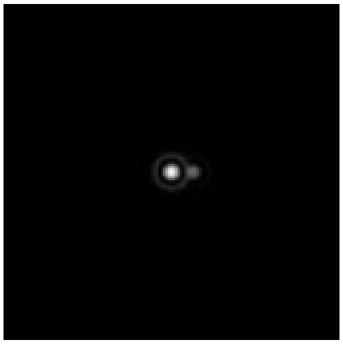
Knowing the quality of your scope and the limits implied by the above formulae will help you solidify your expectations of your scope's performance.

Affects of Color and Magnitude

The dimension of the Airy disk varies with the wavelength of light, being larger for longer wavelengths, red light and smaller for shorter wavelengths, blue light. Whereas the Rayleigh limit is 5.45/D for yellow light at 550nm, for blue light at 450nm it is 4.46/D and for red light at 650nm it is 6.44/D. Therefore, given various colored pairs of equal magnitudes, it would be slightly easier to split two blue stars than two yellow stars and both are easier than two red stars would be. A 6 inch objective that has a common (yellow light) Rayleigh resolution limit of 5.45/D or 0 .9 arcseconds may be able to resolve two blue stars if they are as close as 4.46/D or 0.75 arcseconds. However the same 6 inch scope may only be able to resolve two red stars if they are separated by 6.44/D or 1.1 arcseconds. All the measurements above are for two stars of only moderately bright magnitude, assumed near 6th magnitude.

Rayleigh or Dawes limits usually cannot be reached when viewing doubles that are very bright, have widely varying magnitudes or are very faint. These are more difficult conditions.

Epsilon Bootes, a popular target, is fairly wide in magnitude between the two components. With a difference of about 2.5 mag, it still requires effort at 140x with a 5" SCT to see the fainter secondary component, even though it's 2.8" apart. Doubles with a wide difference in magnitude can be especially troublesome in an obstructed scope where the scope is throwing a larger portion of the light into the first diffraction ring, it tends to sometimes obscure the fainter component. For comparison, on numerous occasions I've split both components of e1e2 Lyra, the Double-Double, both closer in separation but with much closer magnitude components, using 60mm and 76mm scopes and magnifications of 70x to 80x that resulted in apparent separations only between 160 and 200 arcseconds.



Examples of doubles with a wide difference in magnitude:

Name	Con	Prim-Sec / sep	PA	Prim-Sec / sep	PA	Prim-Sec / sep / PA	RA +/- Dec	Note
		SkyCatalogue2000		WDS (most 2002)		6th Orb Cat for 2004		
1 a	UMi	2.0-9.0 / 18.4"	218	2.1-9.1 / 18"	216		RA02h32 +89 16	Σ 93
4	Aur	5.0-8.0 / 5.4">	359o	5.0-8.2 / 6.3"	356	NA	RA04h59 +37 53	Σ 616
sigma2	UMa	4.9-8.2 / 4.0"	353	4.9-8.9 / 3.9"	352		RA09h10 +67 08	Σ 1306
Σ 3057	Cas	6.6-8.7 / 3.7"	299	6.7-9.3 / 3.8"	299		RA0h04.9 +58 32	
Σ 712	Ori	7.5-9.5 / 3.1"	063	6.7-8.6 / 3.1"	065		RA05h26 +02 56	near 30Ori
36 e	Boo	2.5-4.9 / 2.8"	339	2.6-4.8 / 2.8"	344		RA14h45 +27 04	Σ 1877
5	Aur	6.0-9.7 / 3.7"	265	6.0-9.5 / 2.7"	281		RA05h00 +39 24	OΣ 92
Σ 385	Cam	4.2-8.5 / 2.4"	162	4.2-7.8 / 2.5"	159		RA03h29 +59 56	
OΣ 67	Cam	5.3-8.5 / 1.9"	049	5.3-8.1 / 1.7"	049		RA03h57 +61 00	
78iota	Leo	4.0-6.7 / 1.8">	113o	4.1-6.7 / 1.6"	107	4.00-6.81/1.81>/107	RA11h24 +10 32	Σ 1536
2	Vul	5.4-9.2 / 1.8"	127	5.4-8.8 / 1.6"	131		RA19h18 +23 00	B 248
78	Uma	5.1-7.4 / 1.5"<	073	5.0-7.9 / 1.3"	084		RA13h01 +56 22	B 1082

The nearby doubles Struve 3057 and 3062 in Cassiopeia help illustrate the difficulty of a wide difference in magnitude. I have seen both, but I find 3057 the more difficult of the two.

Name	Con	Prim-Sec / sep	PA	Prim-Sec / sep	PA	Prim-Sec / sep / PA	RA +/- Dec	Note

		SkyCatalogue2000		WDS (most 2002)		6th Orb Cat for 2004	
Σ 3057	Cas	6.6-8.7 / 3.7”	299	6.7-9.3 / 3.8”	299		RA00h04.9 +58 32wide
Σ 3062	Cas	6.4-7.2 / 1.5”	335	6.4-7.3 / 1.5”	335		RA00h06.3 +58 26faint

These two double stars are just south of Caph, Beta Cas. The closer pair, 3062 was just barely seen using a 5” at 150x. The wider pair, 3057 was not seen at all in the 5” on several tries, but required a 6” at 200x. Although 3057 is wider, it appears more difficult, probably due to the much wider difference in magnitudes.

The doubles Struve 385, 384 and 400 illustrate a similar wide vs. faint condition. I have seen both 384 and 400, but I have never seen 385, the widest of the group. I have seen Otto Struve 67 only once.

Σ 385	Cam	4.2-8.5 / 2.4”	162	4.2-7.8 / 2.5”	159		RA03h29 +59 56 wide
Σ 384	Cam	7.9-9.1 / 2.0”	270	8.1-8.9 / 2.0”	274		RA03h29 +59 54 faint
OΣ 67	Cam	5.3-8.5 / 1.9”	049	5.3-8.1 / 1.7”	049		RA03h57 +61 00
Σ 400	Cam	6.8-7.6 / 1.6”	265o	6.8-8.0 / 1.4”	266	6.43-7.57/1.45”>/266	RA03h35 +60 02 near vdB14

These doubles are near the bright nebulae vdB14 and vdB15 on the Perseus - Camelopardalis border. Struve 400, 30min east of vdB14, although it is the closest double, is seen more often, usually between 200x and 225x in a 5” scope. OttoStruve 67, even though wider, after many tries was only once suspected (and confirmed) with the 5 inch at 225x.

These examples illustrate the difficulty associated with wide magnitude differences between components. All these observations are true splits. Taking into account the resolution needed to see a black space, most all of these separations should be seen with scopes of 80mm or more. The closer pair in each example, although not of particularly wide difference in magnitudes, being faint, is still difficult. Although less scope would be predicted, they cannot be seen split with anything less than my 5” scope. In one case a 6” was the minimum needed to see a split.

Rasalgethi, alpha Hercules, a fairly wide magnitude double at 3.0-5.4 / 4.6”, required magnification of 60x to see a split even with a 5” scope. It was just barely split with a 3” scope.

It was mentioned that faint stars result in a smaller Airy disk. This might lead you to believe it is easier to resolve faint stars. **Contrary to what you might think, it has been proven more difficult than normal to resolve faint stars.** Based on data reported by Sidgwick, two stars of 9th magnitude might require 50% wider separation beyond the Rayleigh Limit criterion before they can be observed as split. A scope that can resolve two 6th mag stars at 2” may not be able to resolve 8th or 9th mag stars unless they are separated by at least 3”. Resolution is dramatically reduced by diminished light. For a further explanation of this, please refer to Sidgwick.

Examples of faint doubles are listed here:

Name	Con	Prim-Sec / sep	PA	Prim-Sec / sep	PA	Prim-Sec / sep / PA	RA +/- Dec	Note
		SkyCatalogue2000		WDS (most 2002)		6th Orb Cat for 2004		
17	Hyd	6.8-7.0 / 4.1”	002	6.7-6.9 / 4.1”	004		RA08h55 –07 58	Σ 1295
Σ 182	Cas	8.2-8.2 / 3.6”	125	8.3-8.4 / 3.5”	125		RA01h56 +61 16	11’e o.c.663

Σ 559	Tau	6.9-7.0 / 3.1”	277	7.0-7.0 / 3.0”	277		RA04h33 +18 01	mid end V
14	Ori	5.8-6.5 / 0.8”>	314o	5.8-6.7 / 0.8”	315	5.33-6.23/0.84”>/312	RA05h08 +08 30	Σ 98 locate
Σ 643	Ori	8.5-8.5 / 3”	----	9.6-9.6 / 2.4”	304		RA05h08 +08 24	(6’ s 14Ori)
51uBC	Boo	7.0-7.6 / 2.3”<	007o	7.1-7.6 / 2.2”	334	6.51-7.11/2.25”/007	RA15h25 +37 21	Σ 1938
Σ 384	Cam	7.9-9.1 / 2.0”	270	8.1-8.9 / 2.0”	274		RA03h29 +59 54	near vdB14
Σ 3062	Cas	6.4-7.2 / 1.5”	335	6.4-7.3 / 1.5”	335		RA00h06.3 58 26	faint B Cas
Σ 400	Cam	6.8-7.6 / 1.6”	265o	6.8-8.0 / 1.4”	266	6.43-7.57/1.45”>/266	RA03h35 +60 02	near vdB14
20	Dra	7.1-7.3 / 1.4”>	067o	7.1-7.3 / 1.1”	068	6.40-6.64/1.16”/068	RA16h56 +65 00	Σ 2118
A 953	Cas	8.8-8.8 / 0.6”	253	9.1-9.2 / 0.8”	065		RA01h55 +59 55	80’s Σ 182
48	Vir	7.2-7.5 / 0.8”	205	7.1-7.7 / 0.6”	198		RA13h04 –03 40	B929

Sigma 384, a 2” double with components of mag8 and mag9 is very difficult. It has not been seen with less than 225x in a 5”. Alkalurops, 51 Boo is a nice triple with the faint BC components separated by 2.2”. My records show usual magnification is over 200x to see BC split.

For comparison, on numerous occasions I’ve split both components of e1e2 Lyra, the Double-Double, both with much brighter mag5 or mag6 components, using 60mm and 76mm scopes and magnifications of 70x to 80x that resulted in apparent separations only between 160 and 200 arcseconds.

Likewise, stars that are very bright are equally difficult to separate as the above examples.

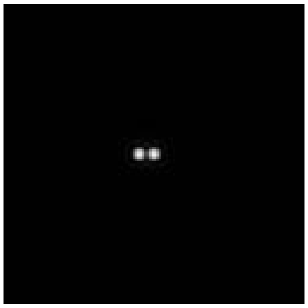
Name	Con	Prim-Sec / sep SkyCatalogue2000	PA	Prim-Sec / sep WDS (most 2002)	PA	Prim-Sec / sep / PA 6th Orb Cat for 2004	RA +/- Dec	Note
41 y	Leo	2.2-3.5 / 4.4”>	125o	2.4-3.6 / 4.5”	125	2.01-3.16/4.43”/125	RA10h20 +19 51	Algieba
66 a	Gem	1.9-2.9 / 4.3”>	065o	1.9-3.0 / 4.1”	064	1.58-2.62/4.19”/061	RA07h34 +31 53	Castor
50 z	Ori	1.9-4.0 / 2.3”	166	1.9-3.7 / 2.5”	165		RA05h41 –01 57	Alnitak
29 y	Vir	3.5-3.5 / 1.5”<	200o	3.5-3.5 / 1.0”<	244	2.74-2.79/0.63”</216	RA12h42 –01 27	Porrima

Castor 1.6-2.6 / 4.2”, was barely split at 60x in the 85mm, but at 66x, it was a clean split. Alnitak 1.9-3.7 / 2.5”, in the 85mm at 120x was not sure, but at 132x it was seen split. At 150x the fainter component looked blue. Algieba 2.0-3.2 / 4.4” was perfectly split at 100x in my small 76mm Schmidt Newtonian catadioptric. At 80x it could barely be seen as double.

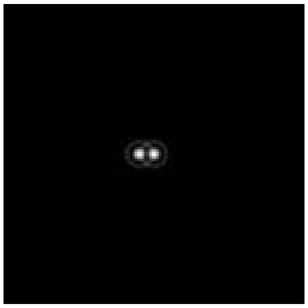
These examples show it took approximately 50% larger apparent size and a larger scope to see bright or faint doubles.

Affects of Central Obstruction

A refractor puts only a small percentage of the total light into the first diffraction ring, and about 85% of the light remains in the central visible disk.



An obstructed scope, depending on the area of the obstruction, may put two or three times as much light into the first diffraction ring while reducing the amount of light in the visible disk resulting in a slightly smaller visible disk.



It will be noted here that a scope with central obstruction of 20% the diameter of the full aperture, even though it will reduce the light in the visible disk and put more light into the first diffraction ring, will have almost no discernable affect on the overall resolution as compared to an unobstructed aperture of the same diameter.

If both scopes were the same size aperture, regardless of f#, the angular size of the Airy disk would be the same. The obstructed scope would have a smaller visible disk while the refractor would have a bigger visible disk and a dimmer first diffraction ring.

An example comparing an unobstructed scope to one with a large central obstruction:
Assume 6" apertures
Rayleigh Limit is 5.45/6 = 0.91 arcsec = radius to minima in first dark interspace.

Diameter across Airy disk is 2 x 0.91 = 1.82 arc seconds in both scopes. The visible disk of a bright star takes up as much as 85% of the diameter of the Airy disk. For a faint star the visible disk may take up only 50% of the diameter. Again, lets use a 60% disk for this example. Then the visible disk measures only 1.82 x 60% =1.09 arcsec diameter. So while the Airy disk measures a diameter of 1.82" arc, the visible disk is a bit smaller.

If a telescope puts more of the light into the first diffraction ring, which is outside the Airy disk, the light left in the visible disk is decreased. We have already established the visible disk is always smaller (can be 85% vs 50% by magnitude) for a fainter star.

If the size of the visible disk were to decrease by just 10% due to light lost into the first diffraction ring due to the obstruction, in this case the size would decrease by 1.09 x 10% = 0.11" from 1.09" to 0.98". But the Airy disk diameter does not change, so the first dark interspace gets wider. While the Airy disk diameter in both scopes remains 1.82”, in the refractor the visible disk may measure 1.09” and in the reflector the visible disk may be as small as 0.98”. Theoretically, you could split closer doubles with the obstructed scope than with the refractor. The values used in this example to illustrate the difference may be somewhat greater than what might actually be achieved, but it does illustrate the example well.

The refractor with less light in the diffraction ring to hide a faint companion may be a better instrument for uneven doubles. The obstructed scope may be better on even doubles because the visible disk is slightly smaller and you might be able to achieve a fraction of an arc second closer split.

Affect of Eye Pupil on Resolution

What are the affects on resolution when eye pupil is smaller than exit pupil? If eye pupil remains larger than exit pupil, then exit pupil controls the amount of light delivered to the eye. The special condition will sometimes occur where eye pupil is smaller than exit pupil and it helps to have an understanding of the implications. There is a general lack of discussion in the texts I have referred too as relates to the application of diffraction theory to the question raised above. Most texts state simply that magnification, exit pupil and focal length have no affect on resolution.

Many texts discuss diffraction and resolution. While some leave the reader groping for complete explanation on which to base conclusions, others give a clear explanation. When it comes to resolution, they all have one thing in common. They all say resolution is dependent only on aperture. It is up to the reader to develop a clear understanding of the concept and apply it correctly.

Few if any texts discuss the affects on resolution when eye pupil is smaller than exit pupil. In the following condition, when using a binocular, if eye pupil is smaller than exit pupil while the magnification does not change, does resolution still remain constant? What else if anything in the optical system might change?

The key concept here is aperture, magnification and exit pupil are inextricably bound together. If exit pupil varies and magnification doesn’t change, something else must change.

In the binocular, magnification is constant. Ignore acuity for a moment and just think laws of optics. There is no disputing the fact that the resolution in the focal plane is delivered by the full aperture. But what happens to that resolution when it is delivered to an eye pupil smaller than the exit pupil? As I vary the eyepiece magnification in my telescopes, the aperture remains constant so exit pupil changes as magnification changes and resolution remains constant. That’s not what happens in binoculars and in some cases telescopes at very low powers. In binoculars, many times entrance pupil can be made smaller by the eye but magnification cannot change. Because of this and the laws of optics, something else is forced to change and that is referred to as “effective aperture.”

As eye pupil gets smaller than exit pupil in binoculars, or any optics, given all other parameters in the system remain constant, one parameter has to vary for the laws of optics to still hold true. We could force aperture to remain constant, but then the only way you could get a smaller exit pupil with a constant magnification is for focal length to change. But with magnification still constant, focal length has not change. Magnification cannot change, so the only other parameter left to change is the “effective aperture.”

A proper application of the laws of optics can give only one result. **If eye pupil is smaller than exit pupil**

while magnification remains constant, only one other parameter of the system can change and that is referred to as "effective aperture." The net affect reduced "effective aperture" has on resolution can then be explained by the laws of optics. Effective aperture is considered that which would provide the equivalent exit pupil that matches the smaller eye pupil and resolution would be based on that effective aperture.

If resolution does change, why is it we may not be able to see this? The difference between a full exit pupil of 5mm and half that (2.5mm eye pupil) for a 50mm binocular observing an object at a distance of 100 feet is 5.5 arcseconds angular, a linear dimension of only 0.03 inches. While this is a very real number, it is not something anyone is likely able to see.

Resolving Power

It has already been explained, the image of point sources are not points, they are disks. They can be so close that they overlap to the extent they cannot be visually perceived as two points. Magnifying may be sufficient to separate these if their images were points, but since the images are disks all it will do is magnify the overlapping disks. In this case, resolution can only be improved by increasing D. This will have the effect of making the disks smaller allowing for less overlap in the image.

Derivation of the formula shows as equivalent the results of the formula for calculating the radius of the Airy disk and the results of the formula for Rayleigh limit. Therefore the Rayleigh limit can be used for a given telescope to calculate the Airy disk radius produced by that telescope. **Any feature that does not have angular dimension greater than the Rayleigh Limit, the limit of the telescope's ability to resolve, cannot be observed as anything smaller than the Airy disk.** Since the Airy Disk is the smallest image spot size that can be achieved by a given telescope, no matter how large that image is magnified, it cannot be construed as having width. You would not be magnifying a resolved feature. You would simply be magnifying the smallest image spot size.

Conversely, any telescope that does not have the ability to produce a resolution finer than the size of the feature will never see that feature as having dimension. A feature may be seen by smaller scopes, especially as we have to consider features other than point sources such as craters on the moon or the Cassini division as having length and width, but the feature is not seen resolved with the width dimension exceeding image spot size unless the telescope resolution is sufficient.

Resolution Limit is derived from the Rayleigh limit criteria, 5.45/D inches or 138/Dmm. The following table lists Dawes and Rayleigh limits for various apertures. Also listed is the expected minimum separations needed to split various combinations of double stars. Rayleigh limit gives the Airy disk radii for telescopes of various diameters D. The diameter of the Airy disk is twice these values, however the visible disk, depending on the magnitude of the object observed, can range from about 85% to less than 50% of the full diameter of the Airy disk. To confirm the ability of your telescope to achieve these predicted limits, it would be necessary to observe and record various results for doubles near and beyond the projected limits for your scope. Obviously, it will vary depending on many factors including quality of scope, the difficulty of the pair selected, the observer's acuity and seeing.

Table Listing Apertures – Limits and Predicted Separations to Split

Aperture	Aperture	Dawes	Rayleigh	Moderate	Bright	Faint	Uneven

Diameter inches	Diameter mm	Limit 116/D arcsec	Limit 138/D arcsec	even pairs arcsec RLx1.1	pairs arcsec +50%	pairs arcsec +50%	pairs arcsec +100%
	60	1.93	2.30	**2.5**	3.8	3.8	5.1
	70	1.66	1.97	**2.2**	3.3	3.3	4.3
3	76	1.52	1.81	**2.0**	3.0	3.0	4.0
	80	1.45	1.73	**1.9**	2.8	2.8	3.8
	85	1.36	1.62	**1.8**	2.7	2.7	3.6
	90	1.29	1.53	**1.7**	2.5	2.5	3.4
	100	1.16	1.38	**1.5**	2.3	2.3	3.0
4	102	1.14	1.36	**1.5**	2.2	2.2	3.0
4.5	114	1.01	1.21	**1.3**	2.0	2.0	2.7
	120	0.97	1.15	**1.3**	1.9	1.9	2.5
5	127	0.91	1.09	**1.2**	1.8	1.8	2.4
	130	0.89	1.06	**1.2**	1.8	1.8	2.3
6	152	0.76	0.91	**1.0**	1.5	1.5	2.0
7	178	0.65	0.78	**0.9**	1.3	1.3	1.7
8	203	0.57	0.68	**0.7**	1.1	1.1	1.5
9.25	235	0.49	0.59	**0.6**	1.0	1.0	1.3
10	254	0.46	0.54	**0.6**	0.9	0.9	1.2

Resolving Power in Extended Objects

Stellar diffraction limits must be understood to apply diffraction to the resolution of extended objects. This is supported by more than one author as noted previously, and again noted here. A good example may be the Cassini division in Saturn’s rings.

Porcellino: “The resolving power of a telescope is applied most often to double stars, but that is not the only area where it is important. It dictates such things as the sharpness of detail visible on a planetary disk …or the moon. …”

The dimension of the Cassini division has been measured and mapped by close satellite imagery. Therefore, consider it to be a 2800-mile wide dark band bounded immediately on either side by the bright light of the A and B rings. If two points of light were considered as being on directly opposite sides of the division, our perception of the view would be of two point sources with a separation of 0.75 arc-seconds between their centers. We would not see a 0.75” space due to the affects of diffraction.

In order to perceive width in the Cassini division, what we are talking about is seeing across the 0.75" gap, the point source separation dimension, and seeing it as having width, not just as a line. If the gap were beyond the resolving ability of your scope, you may see the gap but the gap would be no more than a line. It would not have dimensional width.

Sidgwick: “A bright line of negligible width, crossing a dark area, may be regarded as consisting of a very large number of contiguous points. Each of these will produce its own diffraction pattern, with the result that the image of the line will be thickened by a fringe on either side.”

In Rayleigh limit we have a criteria that provides for a qualified calculation of two objects just touching with a perceivable black space between them. A 9” telescope has a Rayleigh limit of 0.61 arcseconds. Assuming 60% visible diffraction disks, at 0.61 x 1.2 = 0.73” it could just separate objects to appear with a black space between them. Since they are resolved by the scope, this would indicate that any single isolated points bounding a 0.75” separation as viewed through a 9” scope might just be perceived as having dimension in the resolved image of the black separation between them. Other criteria will be shown that account for the fact the division can be seen with smaller scopes, but this may be the smallest scope that actually will allow the observer to perceive width in the division.

A larger aperture will provide a smaller diffraction disk. High contrast will improve the ability to see the division. And finally, an added linear dimension to the feature helps make the image easier to see. All these things will have an affect on the ability to see the image of the division. We do know this; the Cassini division can be seen by scopes with considerably less than 9” diameter.

Extended Object Resolution Criteria

Cassini obviously is viewed by much smaller scopes than point source diffraction would dictate. I believe the primary reason we all see Cassini with smaller scopes is due to its linear dimension.

Sidgwick: “A linear object may stimulate a sufficient number of cones to produce sight even though its width is 20 or 30 times less than the threshold diameter of a spot.”

This above statement refers to the ability to perceive the feature. It does not however indicate that the dimension of the feature can be resolved.

Remember, I've said in my opinion, because it has linear dimension it becomes much easier to see than a point source. Although the dimension across the division would be similar to the resolution limit, the fact that the observed image has a second dimension, a significantly observable length, we get to observe the object in more than one dimension. This allows our eyes to perceive much greater detail. The eye functions more acutely when viewing very high contrast objects and even more-so when viewing linear objects. Visual perception has a great deal to do with explaining why we can see a black line on an apparent white background with an aperture smaller than point source resolution dictates.

Sidgwick: “Diffraction at the edge of a bright area of the image results in blurring of that edge. Encroachment of light from the bright into the dark area causes a dimming of a narrow strip (of the bright area) and the graying of an equal strip (of the dark area).”

This linear feature extends the high contrast image to an appearance of length or an appearance that might be described as width between two bright points stretched out in a linear dimension. The blurring at the edge condition may have the result of making the division appear wider than it is in reality.

Sidgwick gives examples in his book for visual resolution of various stellar and extended conditions. Point source resolution, a white pinpoint on a black background, is explained by the Airy disk formula. In the following example, R represents point source resolution.

For a black spot on a white background, several experiments show results that indicate results or R/2.3 and R/3.

For a single dark line on a light background, similar to the Cassini division, the same experimenters show results of R/3.5 and R/5. Two other specific tests provided results of R/14 and R/15.

The above conditions are found to be easier to resolve than point sources. The next example is found to be more difficult.

For parallel dark lines on a light background, several results indicate a limit of 1.1R to 1.4R.

Diffraction Fringes Interfere with Resolution

In a perfect system, a larger diameter objective imparts a smaller diffraction fringe around every point of light in an image and the result is we can see finer detail. Contrast is the quality that allows seeing more in the detail provided by the rest of the optical system. Observing at magnification below optimum results in a diffraction fringe surrounding each point of light broader than when using an optical system at optimum resolution, even in extended objects.

In the Cassini division, the light gradient and therefore contrast across the boundary of the division is at a maximum. Planetary observers are using their best optical system to observe at optimum resolution therefore the individual observer, thru the use of optimum equipment, by default holds diffraction fringes within the system in use to a minimum.

High contrast boundaries seen with an optical system operating near maximum potential resolution provides us with a view of an object that does not seem to fit the standard low contrast light gradient criteria of the typical extended object. The Cassini division seems to be the perfect example of an extended object that is as far away as we can get from the classical low contrast criteria of an extended object.

Magnification is Necessary to See

The final achievement that needs to be accomplished is to make the image large enough for the eye to perceive. It is not enough to simply have a scope with a large enough diameter to resolve the object. The limiting factor is the eye’s resolution and sufficient magnification must be employed to raise the image to a size that the eye can perceive.

As I stated earlier, no amount of magnification will enable you to resolve a double star that is beyond your aperture’s limit. Likewise, no objective lens will reach the limits of its maximum resolution unless sufficient magnification is employed. With increased image scale higher magnification utilizes more of the ability of the available aperture.

Kitchin: The resolution of the eye is 3 to 5 minutes of arc; therefore a minimum magnification must be utilized to enlarge the image sufficiently to exceed the eye’s resolution. Therefore minimum magnification of about 1300Dmeters or 30Dinches is needed to realize the potential limiting resolution of the optics.

I did encounter a great deal of text and practical discussion relating empirical data gathered over decades that show a significant amount of magnification is required to show all the resolution the telescope is capable of delivering to the eye. In the same practical use discussions, it is readily accepted as magnification increases, contrast of the image thru the eyepiece, as compared to the sky background, is increased. In fact it is this increase in contrast that sometimes allows some objects to even be seen at all.

Liller: The higher the magnification, the more efficient the eye, at least up to a point. A 150mm objective at 100x will see stars more than a magnitude fainter than the same objective at 30x.

Certain objects take magnification better than others. Objects sometimes need the maximum magnification you can get from your instrument in order to resolve them. Sometimes higher magnification is called for to make the most of the telescope's resolution ability. Double stars and planets are examples that can take a lot of magnification.

High magnification is sometimes required to achieve a telescope's resolution limits. The visibility of the detail provided to the eye depends on many factors, such as the kind of detail (point source, linear or circular), the amount of light in the detail (brightest and faintest are more difficult), the relative contrast of that detail (white on black, black on white, faint diffuse or colored), and the visual acuity of the eye at the telescope (varies widely).

Sidgwick: What is the low limit of magnification needed to resolve two points within the capabilities of the objective? If they are not resolved in the focal plane, no magnification will allow the eye to see separation.

Though two points may be resolved in the focal plane, they will not be seen as resolved unless a high enough magnification is employed to allow the eye to perceive the separation. This will vary with the individual user. Point images need to subtend an angle of at least 1 arcmin or the eye cannot see two points. This would yield a magnification of 13Dinches. In the case of stellar images, the value is nearer to 2 or 3arcmin and these yield 24D to 36Dinches. With lower magnifications than these, there is still potential resolved detail in the focal plane image that has not been developed to the point of visibility.

A decrease in the exit pupil to less than 0.85mm leads to a progressively more negative effect on vision. This imposes an upper limit on useful magnification. The resultant 30D provides the maximum useful magnification that can be employed. At this limit, all the gain in resolution has been reached. Of course this upper limit can be exceeded by twice for some objects, such as close doubles stars.

Acuity Determines Magnification

The resolution of the eyes in practical astronomical observation IS NOT 60 arc seconds. Many measurements relative to visual acuity have been analyzed and results can be found at various sites including google (search on visual acuity), C. R. Kitchin's book "Through The Telescope", Jeff Medkeff's website and my own un-scientific testing (search CN Forums for acuity). The resolution of the eye for astronomical observation is more generally found to be in the range of 150 to 200 arc-seconds, while the occasional individual, under the best of circumstances, can obtain results down as low as 120 to 150 arc-seconds. It is a very rare occurrence to obtain results better than that. Sidgwick includes a very clear discussion of the thresholds of human acuity for dark-adapted observing.

In addition to several dozen recorded observations testing for my own visual acuity, I recently solicited visual acuity data from the general astronomical community. Of the dozen or so replies that I got, several good examples stood out. Those observers that reported a specific star, instrument and power provided good data. A few individuals, those who have been recording information for years, although they may not have reported specific data, reported reliable readings.

What was found is most individuals were able to record readings near 180 arcseconds. Very close doubles near the Rayleigh limit required closer to 240 to 300 arcseconds acuity. Only 4 individuals, including myself reported a reading below 160arcseconds. One individual reported that once found at a higher magnification, split stars could then be observed at progressively lower magnifications until down into the range of 120-130 arcseconds. I attempted this on numerous occasions and the best I could achieve splits on several tries was just under 160 arcseconds but never under 150 arcseconds. Maybe this observer was referring to the fact he could still see doubles as elongated or notched.

Assuming the eye is in the most optimistic of the range and it can achieve 3 minutes of resolution or 180 arc seconds, then an image that has a 2" separation of the Airy disks must be magnified by 180" / 2" or 90x to be seen. Generally, features at the limit of resolution require greater magnification as acuity is found to be closer to 240-300 arcseconds (and not the 180arcseconds used in this example). For a 5" scope with a resolution limit of 5.45/5 = 1.09", magnification needed to see a Rayleigh limit double might be more like 240" / 1.09" = 220x, or 300 / 109" = 275x. This will vary with the acuity of the observer.

The formula 1300Dmeters provides a result of the lowest magnification needed to reach the resolution of the telescope. It provides a result achievable by the most optimistic condition. It also assumes the eye is capable of seeing the resolved image at that magnification, which may not always be enough magnification for close doubles. This formula produces a result that shows optimum resolution is achieved at an exit pupil of just less than 1mm, about 0.75mm. Based on field experience, I find the best planetary resolution with my 5" scope is achieved at magnification of 150x to 180x and with my 6" scope around 180x to 220x, exit pupils between 0.83mm and 0.68mm. This agrees well with the formula, however I find the apparent separation needed to see very close doubles at my acuity results in exit pupils of 0.6mm to 0.4mm.

Besides e1e2 Lyra, another telescope double I've used for acuity testing is 75 Hercules. With components of 4.6-5.6 at 4.1", it's not close enough to be pushing the resolution limits of the scopes. That closeness may make acuity more difficult, resulting in a too high value. Two scopes produced similar results showing the double barely split at 43x and 44x and clearly split at 48x. It was not split at all below 43x. This results in maximum acuity of 43 x 4.1 = 176 arcseconds and a clear separation at 48 x 4.1 = 197 arcseconds.

I recently spent a month using binoculars of 10x, 15x, 16x and 20x to find the limit of each binocular by testing the splitting of doubles. I found that at 10x50 the limit was about 20", but at 16x80 or 15x70 the limit was 13" or 14". I had my greatest success with doubles that were close in magnitude. Where the magnitude varied a lot (+2.5) between the primary and the secondary, the difficulty factor increased the limit by 5" to 10".

With my 10x50 Ultraviews I have split doubles at 22" Alya-Ser, 21" 61-Oph and 20" 24-Com. With the 12x50s I split 100-Her 5.9-6.0/14.2" and E2738 Del 6.6-8.7/14.9" appeared elongated. With 15x70s successful targets were E(sigma)2474 Lyr at 16.2" and 100Her at 14.2". With 16x80s I split E2474, 100Her and also E2470 at 13.4".

The 20x80s split the double star (gamma) y 12 Delphinus 4.5-5.5/9.6". The 15x70s show y Del as elongated with proper orientation noticeable. The Oberwerk 20x80s and the Fujinon 16x70s show Struve953 Mon elongated with proper orientation noticeable. E953 Mon is 7.2-7.7/7.1”. These same three binoculars regularly resolve 3 stars of the Trapezium in M42 and on occasion the 4th was suspected in the Fujinon 16x70s and Oberwerk 20x80s. Finally, on an exceptionally still clear night, the AB component of the Trapezium, at 8.7”, was resolved in the 20x80s at full aperture and also at 20x50. The 20x provided just a bit more magnification that allowed easier splitting of close doubles than could be accomplished with the 16x70s or the 15x70s. My quest to attempt ever closer doubles continues.

I think some examples that might be good targets for high powered binoculars are sigma 17CrB at 6” or zeta 7CrB at 6.3”, E(sigma)953 in Mon at 7.1”, Mesartim y Ari at 7.8” or y Del at 9.6”. For smaller binocs try 100 Herc at 14.2”, or a pair of pairs with a bit wider mag. delta E2470 and E2474, the Double’s double in Lyra at 13.4” and 16.2”. Knowing your own visual acuity is an important value for an astronomer. It helps make many decisions at the scope a lot more straightforward.

Summary / Conclusions

While there are several criteria by which we may measure limits of resolution, not all provide for a clear separation between objects. An understanding of the Airy disk formula, Rayleigh limit, and the affects of wavelength and magnitude of light will provide you with the means to determine limits of separation.

Resolution as used in astronomy is commonly measured by angular measure in units of arc seconds. The Airy disk has the same linear size (more or less) for all scopes of a given focal ratio. The Airy disk has the same angular size for all scopes of a given aperture.

Dawes Limit = 4.56/D inches = 116/Dmm. Dawes Limit is the first point at which a double star is elongated enough to suspect the presence of two stars.

Rayleigh Limit = 5.45/D inches = 138/Dmm. Rayleigh Limit is a measure defining the limit at which two components can be clearly identified as separate components. It defines the distance between the centers of two Airy disks where the maximum of one is placed over the minimum of the other. Like Dawes, it is not a measure of separation required to see a black space between components.

The angular radius of the Airy disk out to the first minima is represented as:
$A = 1.22\lambda / D$, where A in radians = 1.22λ (Lambda) / D (Aperture)
A radians = $1.22\lambda / D$ is equivalent to Rayleigh Limit = A arcsec = 5.45/D inches

Aperture and wavelength determine the size of the Airy disk measured to the first minima.
A larger aperture will provide a smaller diffraction disk and greater resolution.
Longer wavelengths of light result in larger Airy disks.

Magnitude has an affect on the size of the central bright visible disk.
Very bright stars make the central disk larger. Very faint stars make the central disk smaller.

Telescope type has an affect on the size of the central bright visible disk.
Obstructed scopes make the central disk smaller and the first diffraction ring brighter.

The visible disk itself is slightly smaller than the Airy disk dimension since the Airy disk is measured out to the minima of the first dark space. The visible disk varies with magnitude, but it's about 85% of the Airy disk dimension for a bright star. For a faint star just less than 50% of the diameter of the Airy disk is occupied by the central bright disk.

If two stars with separation at the Rayleigh limit were faint and each had a visible disk just a hair less than 50% the diameter of the Airy disk, you would be able to see a thin line of separation.

Assuming a visible disk 60% of Airy disk diameter, a separation of 20% more than the Rayleigh Limit is needed to have a black space between two stars. In order to cleanly split doubles, two stars would need to be separated by a minimum of A arcsec x 60%/50%.

The dimension of the Airy disk varies with the wavelength of light, being larger for red light and smaller for blue light. Whereas the Rayleigh limit is 5.45/D for yellow light at 550nm, for blue light at 450nm it is 4.46/D and for red light at 650nm it is 6.44/D.

A refractor puts only a small percentage of the total light into the first diffraction ring, and about 85% of the light remains in the central visible disk. An obstructed scope, depending on the area of the obstruction, may put two or three times as much light into the first diffraction ring while reducing the amount of light in the visible disk resulting in a slightly smaller visible disk.

If eye pupil is smaller than exit pupil while magnification remains constant, only one other parameter of the system can change and that is referred to as “effective aperture.” Effective aperture is considered that which would provide the equivalent exit pupil that matches the smaller eye pupil and resolution would be based on that effective aperture.

Any feature that does not have angular dimension greater than the limit of the telescope's ability to resolve cannot be observed as anything smaller than the Airy disk. Since the Airy Disk is the smallest image spot size that can be achieved by a given telescope, no matter how large that image is magnified, it cannot be construed as having width. You would not be magnifying a resolved feature. You would simply be magnifying the smallest image spot size.

A telescope with an aperture that will produce an Airy disk smaller than the observed feature is required to see the feature as having dimension. A smaller telescope may observe a feature, but will not have the resolution required to see the feature as dimensional.

An added linear dimension to a feature helps make the extended image easier to see.

No objective lens will reach the limits of its maximum resolution unless sufficient magnification is employed. With increased image scale higher magnification utilizes more of the ability of the available aperture. The telescope will realize full potential resolution with a magnification of approximately 1300Dmeters. This is near the same as 30Dinches.

Some observers may achieve eye resolution of 3 arc minutes or 180”, in which case they could split a 2” double at 90x magnification. Using the same scope, another observer with a greater or lesser acuity will require a different magnification to see the same double. Acuity is not a measure of the ability to resolve in the scope. It is a measure of the ability of the eye to see the resolved image. Objects at the limit of resolution will require a magnification that produces a larger apparent size than the observer's best acuity, possibly in the range of 4 arcminutes.

While acuity will require objects at the limit of resolution to be highly magnified, acuity can be tested on objects with all dimensions of separation, even wide objects that require only low magnification. The observer will find a fairly narrow range of acuity can be determined using doubles with a variety of separations and all manner of equipment, including binoculars.

Credits

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Double Star Data Resources:

Highly magnified diffraction and Airy disk images generated by Abberator, a freeware software written by Cor Berrevoets, Ritthem, The Netherlands, http://aberrator.astronomy.net/index.html

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